HARD INTEGRAL

$$\int_0^{\pi} \frac{2\overset{u}{\cancel{x}}^{3} - 3\pi \overset{u}{\cancel{x}}^{2}}{(1+\sin(\overset{u}{\cancel{x}}))^2}\, d\cancel{x}\,u$$

let $x = \pi/2 - \cancel{x}\,u$

$$\int_{-\pi/2}^{\pi/2} \frac{2(\pi/2 - x)^3 - 3\pi(\pi/2 - x)^2}{(1+\cos(-x))^2}\, dx$$

This is a symmetric integral so $\int_{-a}^{a} f(x)\,dx$

$$= \int_{-a}^{a} \frac{f(x)+f(-x)}{2}\, dx$$

$$\frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{2\left(\left(\frac{\pi}{2}-x\right)^3 + \left(\frac{\pi}{2}+x\right)^3\right) - 3\pi\left(\left(\frac{\pi}{2}-x\right)^2 + \left(\frac{\pi}{2}+x\right)^2\right)}{(1+\cos(x))^2}$$

cos(x) doesn't change as it is even.

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{2(\pi)\left(\left(\frac{\pi}{2}-x\right)^2 - \left(\frac{\pi^2}{4} - x^2\right) + \left(\frac{\pi}{2}+x\right)^2\right) - 3\pi\left(\left(\frac{\pi}{2}-x\right)^2 + \left(\pi/2 + x\right)^2\right)}{(1+\cos(x))^2}\, dx$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{-2\pi\left(\frac{\pi^2}{4} - x^2\right) - \pi\left(\frac{\pi^2}{4} \cancel{- \pi x} + x^2 + \frac{\pi^2}{4} \cancel{+ \pi x} + x^4\right)}{(1+\cos(x))^2}$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{-\frac{\pi^3}{2} + 2\pi x^2 - \pi\left(\frac{\pi^2}{2} + 2\cancel{\pi} x^2\right)}{(1+\cos(x))^2}\, dx$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \frac{-\pi^3}{(1+\cos(x))^2}\, dx$$

$$= \frac{-\pi^3}{2}\int_{-\pi/2}^{\pi/2} \frac{1}{(1+\cos(x))^2}\, dx$$

Using weierstross sub we have

$$\underbrace{\frac{-\pi^3}{2}\int_{-1}^{1} \frac{2}{1+t^2}\cdot\frac{1}{\left(1+\frac{1-t^2}{1+t^2}\right)^2}\, dt}$$

$$\int_{-1}^{1} \frac{2}{\cancel{1+t^2}}\cdot\frac{(1+t^2)^2}{2^2}\, dt$$

$$= \int_{-1}^{1} \frac{1+t^2}{2}\, dt$$

$$= \frac{1}{2}\left[\frac{t^3}{3}+t\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{4}{3}-\left[\frac{-4}{3}\right]\right)$$

$$= \boxed{\frac{4}{3}}$$

$$\therefore \frac{-\pi^3}{2}\left(\frac{4}{3}\right) = \boxed{-\frac{2}{3}\pi^3}$$