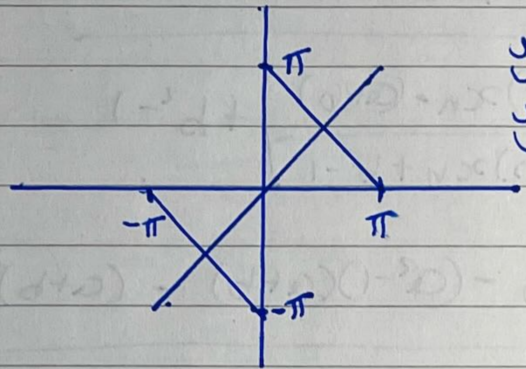
STEP II 2017

Q3.



$y = x$
$y = \pi - x$
$y = -\pi - x$

$$\sin(y) = \frac{1}{2}\sin(x)$$

$$y = \arcsin\left(\frac{1}{2}\sin(x)\right)$$

hence $y' = \dfrac{1}{\sqrt{1-\left(\frac{\sin(x)}{2}\right)^2}} \cdot \dfrac{\cos(x)}{2}$

$$= \frac{1}{\frac{1}{2}\sqrt{4-\sin^2(x)}} \cdot \frac{\cos(x)}{2}$$

$$= \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}$$

hence $\cos(x)\,(4-\sin^2(x))^{-1/2}$

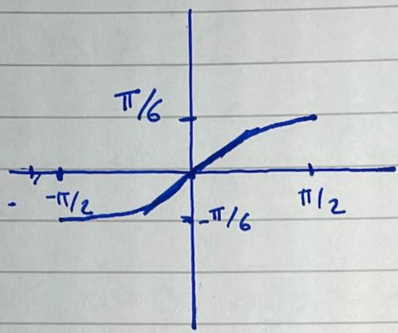
$y'' \rightarrow$ $-\sin(x)$ $\quad -\cancel{\tfrac{1}{2}}(4-\sin^2(x))^{-3/2}(-\cancel{2}\sin(x)\cos(x))$

$$\therefore y'' = \frac{\sin(x)\cos^2(x)}{(4-\sin^2(x))^{3/2}} - \frac{\sin(x)(4-\sin^2(x))}{(4-\sin^2(x))^{3/2}}$$

$$= \frac{\sin(x)(1-\sin^2(x)) - 4\sin(x) + \sin^3(x)}{(4-\sin^2(x))^{3/2}}$$

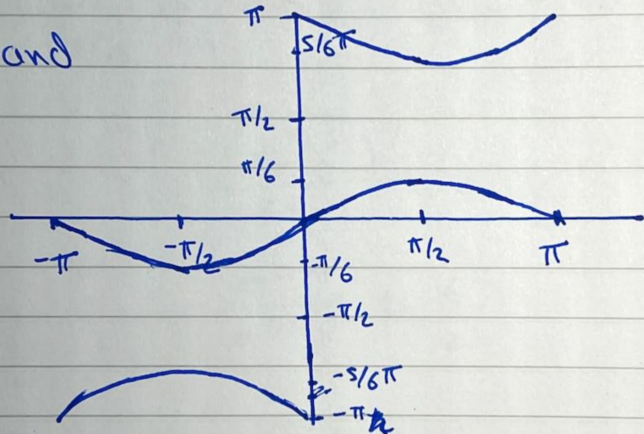
$$= \frac{-3\sin(x)}{(4-\sin^2(x))^{3/2}}$$

$\sin(y) = \frac{1}{2}\sin(x)$



initial gradient of $\frac{1}{2}$ and approaches 0 at $x = \pm \pi/2$.

and



$0 < \sin(y) < 1/2$

$\therefore 0 < y < \pi/6$

and $5\pi/6 < y < \pi$

crosses $\left(\frac{\pm\pi}{2}, \pm \pi/6\right)$ and $(\pi, 0)$ arriving at gradient of $-1/2$

(iii) $\cos(y) = \sin(\pi/2 - y)$

$\cos(y) = \frac{1}{2}\sin(x)$