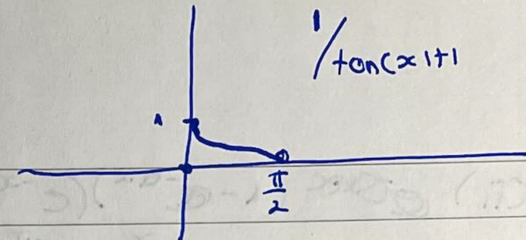


Q3.

let $f(x) = 1/\tan(x)+1$

$0 \leq x \leq \pi/2$

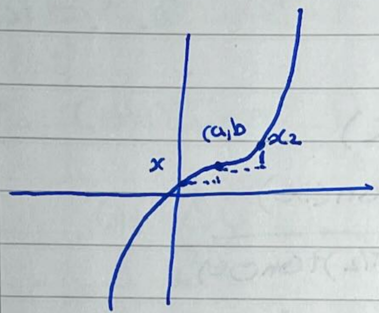
$$f'(x) = \left((\tan(x)+1)^{-1}\right)'$$
$$= -1(\tan(x)+1)^{-2}(\tan(x)+1)'$$
$$= \frac{-\sec^2(x)}{(\tan(x)+1)^2}$$
$$= -\frac{1/\cos^2(x)}{\left(\frac{\sin(x)+\cos(x)}{\cos(x)}\right)^2}$$
$$= -\frac{1/\cos^2(x) \cdot \cos^2(x)}{(\sin(x)+\cos(x))^2}$$
$$= -\frac{1}{1+2\sin(x)\cos(x)}$$
$$= -1/1+\sin(2x)$$

and $-1 \leq f'(x) \leq -1/2$.

(ii) for a graph to have rotational symmetry around a point, $\Leftrightarrow$ it must be an odd function with symmetry around $x=a$

for example, if we pick two points x_1, x_2 and $x_1 = a$ then $x_2 = a$
if $x_1 = 0$ then $x_2 = 2a$
and thus $x_2 = 2a - x_1$ as there is a symmetry around $x=a$

$\therefore$ $g(x_2) = -g(x_1)$ as it is ~~even~~ odd function around $x=a$



let $g(x_2) = b+k$ where k is a constant

thus by rotational symmetry

$\iff g(x_1) = b-k$

$\therefore g(x_2) \iff -g(x_1)$

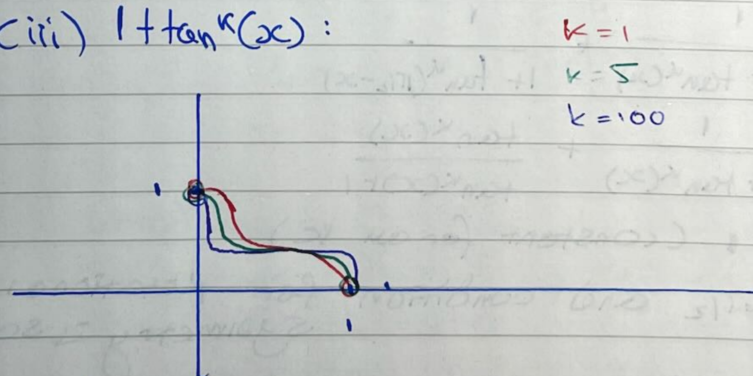
$\therefore g(2a-x) \iff g(x)$

$\therefore g(2a-x)+g(x) = b+k+b-k$

$= 2b$ as required

$$\int_{-1}^{1} g(x)\,dx = 0$$

(iii) $1+\tan^k(x)$:



The point of symmetry is around the inflection point which stays constant regardless of k (graphically shown)

$\therefore f'(x) = \dfrac{-1}{1+\sin(2x)}$ for $k=1$

$\therefore f''(x) = \dfrac{2\cos(2x)}{(1+\sin(2x))^2} = 0 \quad \therefore 2\cos(2x) = 0.$

hence $x = \pi/4$

Since $a = \pi/4$

$\therefore \tan(2a - x) = \tan(\pi/2 - x)$

$$= \frac{\tan(\pi/2) - \tan(x)}{1 + \tan(\pi/2)\tan(x)}$$

$$= \frac{\frac{1}{\cos(\pi/2)} - \tan(x)}{1 + \frac{\tan(x)}{\cos(\pi/2)}}$$

$$= \frac{\frac{1 - \tan(x)\cos(\pi/2)}{\cancel{\cos(\pi/2)}}}{\frac{\cos(\pi/2) + \tan(x)}{\cancel{\cos(\pi/2)}}}$$

$$= \frac{1}{\tan(x)}$$

hence $\frac{1}{1+\tan^k(x)} + \frac{1}{1+\tan^k(\pi/2 - x)}$

$\Rightarrow \frac{1}{1+\tan^k(x)} + \frac{\tan^k(x)}{\tan^k(x)+1}$

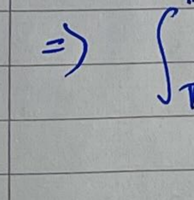
$\Rightarrow 1$ (constant for all x)

hence $b = 1/2$ and condition for Rotational symmetry is satisfied.

$\therefore \int_{1/6\pi}^{1/3\pi} \frac{1}{1+\tan^k(x)}\,dx$

$\Rightarrow \int_{\pi/4 - \pi/12}^{\pi/4 + \pi/12} \frac{1}{1+\tan^k(x)}\,dx$ and since symmetry around point

$\Rightarrow \frac{\pi}{6}\left(\frac{1}{2}\right) = \pi/12$



b's cancel out