Q4.

Since $p(x)-1$ is divisible by $(x-1)^5$ $\therefore$ $\underline{p(1)=1}$

$p(x)-1$ is divisible by $(x-1)^5$

hence $(x-1)^5 q_1(x) + 1 = p(x)$

where $q_1(x)$ is a polynomial of degree 4.

hence $p'(x) = ((x-1)^5 q(x) + 1)'$

$= (x-1)^5 q_1'(x) + 5(x-1)^4 (q_1(x))$

$= (x-1)^4 ((x-1)q_1'(x) + 5 q_1(x))$

hence $(x-1)^4 \mid p'(x)$

(iii) $p(x)+1$ is divisible by $(x+1)^5$

then $p(x) = (x+1)^5 q_2(x) - 1$

where $q_2(x)$ is a polynomial of degree 4

$\therefore p'(x) = (x+1)^5 q_1'(x) + 5(x+1)^4 q_1(x)$

$= (x+1)^4 ((x+1)q_1'(x) + 5q_1(x))$

hence $p'(x)$ is also divisible by $(x+1)^4$

$\therefore$ $p'(x) = m(x+1)^4(x-1)^4$

where m is some constant

hence $p(x) = m\left(\int (x^2-1)^4 \, dx\right)$

$= m\left(\int x^8 - 4x^6 + 6x^4 - 4x^2 + 1 \, dx\right)$

$= m\left(\frac{x^9}{9} - \frac{4}{7}x^7 + \frac{6}{5}x^5 - \frac{4}{3}x^3 + x + c\right)$

$p(1) = 1 = m\left(\frac{1}{9} - \frac{1}{7} + \frac{6}{5} - \frac{4}{3} + 1 + c\right)$

$-1 = m\left(-1/9 + 1/7 - 6/5 + 4/3 - 1 + c\right)$

$$\therefore m = \frac{1}{\frac{1}{9}8 - 1/7 + 6/5 - 4/3 + 1}$$